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Superstring field theory equivalence: Ramond sector

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ABSTRACT: We prove that the finite gauge transformation of the Ramond sector of the modified cubic superstring field theory is ill-defined due to collisions of picture changing operators.

Despite this problem we study to what extent could a bijective classical correspondence between this theory and the (presumably consistent) non-polynomial theory exist. We find that the classical equivalence between these two theories can almost be extended to the Ramond sector: We construct mappings between the string fields (NS and Ramond, including Chan-Paton factors and the various GSO sectors) of the two theories that send solutions to solutions in a way that respects the linearized gauge symmetries in both sides and keeps the action of the solutions invariant. The perturbative spectrum around equivalent solutions is also isomorphic.

The problem with the cubic theory implies that the correspondence of the linearized gauge symmetries cannot be extended to a correspondence of the finite gauge symmetries. Hence, our equivalence is only formal, since it relates a consistent theory to an inconsistent one. Nonetheless, we believe that the fact that the equivalence formally works suggests that a consistent modification of the cubic theory exists. We construct a theory that can be considered as a first step towards a consistent RNS cubic theory.

KEYWORDS: String Field Theory, Superstrings and Heterotic Strings

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1 Introduction

There are two familiar versions of (RNS covariant open) superstring field theory.¹ The first to be constructed is the modified cubic theory of [2–4], in which the NS string field carries zero picture number. This is an improved version of Witten’s theory [5], in which collisions of picture changing operators destroy the gauge symmetry and invalidate the evaluation of scattering amplitudes [6]. Despite correcting the problems of collisions of picture changing operators, some difficulties still remain. Most familiar is the criticism against the appearance of the picture changing operator Y_{-2} in the definition of this theory, on the ground that this operator possesses a non-trivial kernel. It is not clear to us if this actually poses a problem, since one can claim that the problematic string fields should not be considered as legitimate ones.² However, there is also a genuine problem with the cubic formulation: Its Ramond sector is inconsistent, since its linearized gauge transformations cannot be exponentiated to give finite gauge transformations, due to (again) collisions of picture changing operators. This fact was unnoticed so far. We prove it in section 2.

The second formulation is the non-polynomial theory, constructed by Berkovits [8]. This theory lives in the large Hilbert space of the fermionized RNS superghosts [9]. The

¹See [1] for a review of recent developments in string field theory. More detailed background on the construction of superstring field theories can be found in section 8 therein.

²See the companion paper [7] for a more detailed discussions on this subject.

physical degrees of freedom behave like ξ times the usual RNS degrees of freedom. Hence, the NS string field carries ghost number zero (instead of one) and picture number zero (instead of the “natural” minus one picture). A novel gauge symmetry is introduced in order to reduce the degrees of freedom by half, that is, in order to obtain the same amount of degrees of freedom one has in the small Hilbert space.

It was recently discovered that a mapping exists between these two theories, sending solutions of the one to solutions of the other, such that gauge orbits are sent bijectively to gauge orbits [10]. The cohomologies around solutions and the actions of solutions are invariant under this mappings. Moreover, it was also shown in [10] that this formalism can be extended to include also the NS- sectors of both formulations [11, 12]. It was concluded that the theories are classically equivalent.

It should be noted that (in one direction) the mapping is performed by a mid-point insertion on top of the field of the cubic theory. Furthermore, a regularization had to be employed for the evaluation of the action under the mapping and while the regularization was not explicitly constructed, it was explained what should its properties be (symmetry arguments). This state of affairs brings about two possible points of view for interpreting the results of [10]. One option is that the mid-point insertion is a singular limit of another family of mappings, which are more complicated but regular. If this is the case then the theories are genuinely equivalent. The other option is that the cubic theory should be thought of as a singular gauge limit of the non-polynomial theory. Being a singular limit does not imply that it is wrong. It only implies that one has to use some care when working with it. For example, it was realised [13–15] that Schnabl’s gauge [16] is a singular limit of a regular family of “linear b -gauges”. Nonetheless, it was also shown that it is possible to regularize expressions in this gauge and get a reliable final result.

There are at least three obvious matters that should be dealt with regarding the equivalence of the two theories. We already mentioned the somewhat singular nature of the equivalence. The second issue is the inclusion of the Ramond sector in the equivalence, while the third is the extension of the mapping to the quantum level. In fact, the second is a pre-requirement of the third, since Ramond states can be produced in loops even when calculating processes involving only NS fields as external states. However, as we already mentioned, the Ramond sector of the cubic theory is inconsistent. Hence, a genuine correspondence cannot exist. Nonetheless, we prove the existence of a formal equivalence between the theories in section 3. Then, in section 4, we extend the formalism to describe arbitrary brane systems.

We believe that the cubic theory with the Ramond sector should be thought of as being a singular limit, or a bad gauge fixing, of another, benign theory. One candidate for the consistent theory is the non-polynomial theory. However, the fact that the formal manipulations performed in section 3 show that (up to mid-point regularization issues) the cubic and non-polynomial theories are formally equivalent, seems to suggest that regardless of its inconsistency in its current formulation, not all is bad in the cubic theory. A possibility for defining a consistent cubic theory is discussed in section 5, where we study a theory with two Ramond string fields that has some attractive features. Nonetheless, since in this theory the Ramond sector is doubled, it should somehow be further modified in order

to give the cubic theory that we seek. We did not find a way to do that in a manner that avoids the infamous picture changing collisions. Hence, this theory is only a first step towards the goal of a consistent cubic theory. We comment on that and on some other issues in the conclusions section 6.

In the non-polynomial theory, the incorporation of the Ramond sector was performed in [8, 17, 18]. While this theory seems to be consistent, it is not known how to define it covariantly using a single Ramond string field. The best one can do covariantly is to write an action with two Ramond fields, at pictures $\pm\frac{1}{2}$ and add a constraint relating them. One could try to implement the constraint using a Lagrangian multiplier. However, this will introduce a non-trivial equation of motion for the new Lagrangian-multiplier-string-field. It might be possible to get around this issue by making this field a part of a quartet or by adding more Lagrangian multipliers along the lines of [19], but this was not done so far.

The parity of the Ramond string fields in both theories is the same as that of the NS string fields of the same theory. This might come as a surprise, since the vertex operators in the Ramond sector have an opposite parity to that of the NS+ sector. However, since the components of the Ramond string field represent fermions in space-time, the coefficient fields themselves have to be odd. This brings the total parity of the Ramond string field to the desired value, i.e., they are odd in the cubic theory and they are even in the non-polynomial one. We refer the reader to appendix A for some tables of the quantum numbers of the relevant operators and string fields.

We end this introduction by presenting some conventions and some of the operators we use. Throughout this paper $[A, B]$ denotes the graded commutator, i.e.,

$$[A, B] \equiv AB - (-)^{AB}BA, \tag{1.1}$$

where A and B in the exponent represent the parity of A and B . An important building block in our construction is the operator,

$$P(z) = -c\xi\partial\xi e^{-2\phi}(z). \tag{1.2}$$

This operator is a contracting homotopy operator for Q in the large Hilbert space, i.e.,

$$[Q, P(z)] = 1. \tag{1.3a}$$

Other important relations are,

$$[Q, \xi(z)] = X(z), \tag{1.3b}$$

$$[\eta_0, \xi(z)] = 1, \tag{1.3c}$$

$$[\eta_0, P(z)] = Y(z), \tag{1.3d}$$

where X and Y are the picture changing operator and the inverse picture changing operator respectively. These operators obey the OPE,

$$XY \sim 1. \tag{1.4}$$

It is also possible to define the double picture changing operators X_2 and Y_{-2} , obeying,

$$Y_{-2}X \sim Y, \quad X_2Y \sim X, \quad X_2Y_{-2} \sim 1. \quad (1.5)$$

We will also use the following (regular) OPE's,

$$PX \sim \xi, \quad (1.6a)$$

$$\xi Y \sim P, \quad (1.6b)$$

$$PP \sim 0, \quad (1.6c)$$

$$\xi\xi \sim 0, \quad (1.6d)$$

$$P\xi \sim 0. \quad (1.6e)$$

The divergent OPE's are XX , YY , $X\xi$ and YP .

We will use the operators appearing in (1.6) as mid-point insertions on string fields. This is not a-priori excluded, since they are all primaries of zero conformal weight [7]. When these operators are inserted at the mid-point over the string fields $\Psi_{1,2}$ having no other mid-point insertions one finds,

$$(\mathcal{O}_1\Psi_1) \star (\mathcal{O}_2\Psi_2) = (-)^{\mathcal{O}_2\Psi_1}(\mathcal{O}_1\mathcal{O}_2)(\Psi_1 \star \Psi_2) = (-)^{\mathcal{O}_2(\mathcal{O}_1+\Psi_1)}(\mathcal{O}_2\mathcal{O}_1)(\Psi_1 \star \Psi_2), \quad (1.7)$$

that is, the star algebra factorizes to a product of a graded-Abelian mid-point operator-insertion algebra and a regular string field star algebra. We shall often refer to (1.6), when we actually mean its part within expressions of the form of (1.7). Henceforth, we shall omit the star product, as it is the only possible product of string fields.

2 The Ramond sector of the cubic theory is inconsistent

Witten's cubic superstring field theory [5] was shown to be inconsistent, due to singularities in its gauge transformations [6, 20]. These singularities emerge from collisions of picture changing operators: The picture changing operator X is inserted at the string mid-point, which is invariant under the star product. Hence, the double pole in the XX OPE gives rise to infinities when the linearized gauge transformation is plugged into the action. The origin of this problem lies in the presence of the picture changing operator X in the linearized gauge transformation in Witten's theory,

$$\delta A = Q\Lambda + X[A, \Lambda]. \quad (2.1)$$

This in turn is inevitable, since the NS string field A and the NS gauge string field Λ are both of picture number -1 . Changing the picture of Λ to zero, would force one to introduce an insertion of the inverse picture changing operator Y on top of the $Q\Lambda$ term and collisions would still occur, now due to the double pole in the YY OPE.

In order to remedy this problem, it was suggested that the NS string field A should have a zero picture number [2-4]. The Ramond sector is then described by the picture $-\frac{1}{2}$

string field α [2]. The modified action reads,

$$S = S_{\text{NS}} + S_R, \tag{2.2a}$$

$$S_{\text{NS}} = - \int Y_{-2} \left(\frac{1}{2} A Q A + \frac{1}{3} A^3 \right), \tag{2.2b}$$

$$S_R = - \int Y \left(\frac{1}{2} \alpha Q \alpha + A \alpha^2 \right), \tag{2.2c}$$

and its equations of motion are,

$$Y_{-2}(Q A + A^2) + Y \alpha^2 = 0, \tag{2.3a}$$

$$Y(Q \alpha + [A, \alpha]) = 0. \tag{2.3b}$$

Acting on these equations with X_2, X respectively brings them to the form

$$Q A + A^2 + X \alpha^2 = 0, \tag{2.4a}$$

$$Q \alpha + [A, \alpha] = 0, \tag{2.4b}$$

where (1.4) and (1.5) were used.

The action (2.2) is invariant under the following infinitesimal gauge transformations,

$$\delta A = Q \Lambda + [A, \Lambda] + X[\alpha, \chi], \tag{2.5a}$$

$$\delta \alpha = Q \chi + [\alpha, \Lambda] + [A, \chi], \tag{2.5b}$$

where Λ and χ are the NS and Ramond gauge string fields respectively. It is clear that no collisions can emerge when only the NS sector is considered, since no picture changing operators appear in this case.

In [2], it was claimed that this gauge transformation is regular also in the Ramond sector. To “prove” that, the infinitesimal transformation (2.5) was plugged into the equations of motion (2.4). It was found that this does not lead to collisions of picture changing operators at the leading order in the gauge string fields. However, if one considers also the term quadratic with respect to the gauge string fields, collisions do occur. One may hope that adding the next (first non-linear) order to (2.5) produces another singular term that cancels the former. This is plausible a-priori, since both terms are quadratic with respect to χ . Nonetheless, a direct evaluation reveals that this is not the case. Moreover, the singularity of the transformation can be seen even without referring to the action. Consider for simplicity the case $A_0 = 0, \alpha = \alpha_0 \neq 0$ and act with a gauge transformation in the Ramond sector, i.e., take $\Lambda = 0$ and $\chi \neq 0$. Further, assume for simplicity that $Q \chi = 0$. Explicit iterations of the linearized gauge transformations give,

$$\alpha \rightarrow \alpha_0 \rightarrow \alpha_0 + X[[\alpha_0, \chi], \chi] \rightarrow \dots, \tag{2.6a}$$

$$A \rightarrow X[\alpha_0, \chi] \rightarrow 2X[\alpha_0, \chi] \rightarrow 3X[\alpha_0, \chi] + X^2[[[\alpha_0, \chi], \chi], \chi]. \tag{2.6b}$$

We see that the expression we get for A after the third iteration is generically divergent. Furthermore, by plugging this expression into the action and expanding it to second order with respect to χ we also obtain divergences, as stated.

One may think that iterating the linearized transformation, as we do here, is too naive and that the full non-linear transformation will somehow manage to avoid this problem. This is not the case. The full non-linear transformation is obtained from iterating the linearized one, while rescaling the gauge fields at the n^{th} iteration as

$$\Lambda \rightarrow \frac{\lambda}{n}\Lambda, \quad \chi \rightarrow \frac{\lambda}{n}\chi, \quad (2.7)$$

where λ is a fixed parameter. This fixes the coefficient of the linearized transformation to λ , while assuring the infinitesimal character of the gauge field. Hence, the only difference between the expression above and the exact one is in the numerical values of the coefficients, which will not change dramatically. Nonetheless, in order to remove any doubts let us consider also the full non-linear transformation, pretending for the moment that it exists. The differential equation defining it is of the form,

$$\frac{d}{d\lambda}\vec{A}(\lambda) = V + \mathcal{L}\vec{A}(\lambda). \quad (2.8)$$

Here, we defined,

$$\vec{A}(\lambda) \equiv \begin{pmatrix} A(\lambda) \\ \alpha(\lambda) \end{pmatrix}. \quad (2.9)$$

Now, λ serves as an evolution parameter for the gauge transformation and the initial condition $\vec{A}(0)$ is the string field before the gauge transformation. The fixed string field V and linear operator \mathcal{L} in our case are,

$$V = \begin{pmatrix} Q\Lambda \\ Q\chi \end{pmatrix}, \quad (2.10)$$

$$\mathcal{L} = \begin{pmatrix} \Lambda_R - \Lambda_L & X(\chi_R - \chi_L) \\ \chi_R - \chi_L & \Lambda_R - \Lambda_L \end{pmatrix}, \quad (2.11)$$

where Λ_R and χ_R represent multiplication from the right by Λ or χ , while Λ_L and χ_L operate from the left. Note, that left and right operations commute. The general solution of (2.8) is,

$$\vec{A} = e^{\lambda\mathcal{L}}\vec{A}(0) + \frac{e^{\lambda\mathcal{L}} - 1}{\mathcal{L}}V, \quad (2.12)$$

where the operator multiplying V is defined by its Taylor series and is regular. It is easy to verify that in the pure NS case (2.12) reduces to the familiar form,

$$A(\lambda) = e^{-\lambda\Lambda}(A(0) + Q)e^{\lambda\Lambda}. \quad (2.13)$$

For the example considered in (2.6), we see that the divergent term $X^2[[[\alpha_0, \chi], \chi], \chi]$ indeed appears and its coefficient is $\frac{\lambda^3}{6}$. Higher order terms have higher degree of divergence, i.e., higher powers of X , multiplying other conformal fields and the total expression has the form of an essential singularity.

A way out could have still existed. The powers of X in (2.12) are partially correlated with the power of the gauge string field χ . Hence, constraining χ to always carry a factor of

Y in its definition could potentially eliminate the singularity. However, this will introduce singularities due to collisions of Y in (2.12). In fact, in this case singularities will emerge even earlier, since the action also contains a factor of Y . An operator that could have worked is P , since its OPE with X give ξ , which has a trivial OPE with P and on the other hand, no singularities can emerge from iterations of P itself (1.6). Nonetheless, this cannot be an accepted resolution, since P and ξ do not live in the small Hilbert space, to which \vec{A} belongs. Replacing $P(i)$ by, say, $P(i) - P(0)$ that does live in the small Hilbert space would not solve the problem, since some of the singularities would still be left. The only option for eliminating the singularities in such a way would be to use $P(i) - P(-i)$. This, however, does not resolve the problem of singularities in the action, since the YP OPE diverges. Moreover, if one constrains the gauge string field to contain some given mid-point insertion, it seems to us that the physical string field should also be constrained in a similar way, since otherwise the linearized equation of motion would not correspond to the world sheet theory results. Constraining the physical string field to have some insertions as part of its definition essentially leads to a redefinition of the theory. The redefined theory can be Witten's theory or some other variant, e.g., [21]. However, it seems that there is no simple modification of the mid-point structure that resolves the singularities both in the gauge transformation and in the action.

Another possible resolution would be to replace the X in the definition of the linearized gauge transformation by some sort of an operator that will effectively induce a projection to the correct space of string fields.³ It might be the case that a variant of the regularized X insertions of [2] would work. These variants are non-local operators, that were introduced in order to resolve the singularities in the tree-level scattering amplitudes. Presumably, if they are good for resolving one sort of a singularity of the theory they might also help to resolve the other. However, for the sake of the linearized gauge transformation, the mid-point character of the X insertion is important. Specifically, the relation,

$$X(\Psi_1\Psi_2) = (X\Psi_1)\Psi_2 = \Psi_1(X\Psi_2), \tag{2.14}$$

is imperative for proving that this is actually a symmetry. It is not clear to us how could a non-local variant of X obey this relation. Also, one would still have to verify that the resulting (finite) gauge symmetry is singularity-free. These points would have to be addressed in any attempt to resolve the gauge symmetry singularities along these lines. Such a resolution, if possible at all, would be a non-trivial redefinition of the gauge symmetry.

We conclude that, for the current formulation of the theory, the well-defined (up to issues regarding the space of string fields) linearized gauge transformation cannot be exponentiated to a finite gauge transformation. Hence, the fermionic gauge invariance is lost. One might be tempted to give up this invariance. However, this would result in a wrong number of fermionic degrees of freedom, the theory is then no longer open string theory and there is no reason to believe that it would be well defined quantum mechanically. Hence, the Ramond part of the cubic action in its current form cannot be trusted.

³This possibility was proposed to me by Scott Yost.

3 The equivalence

Regardless of the inconsistency of the cubic theory, we would like to examine to what extent could the NS sector equivalence of [10] be extended to the Ramond sector. We manage to construct mappings between solutions of both theories in 3.1 and we prove in 3.2 that (up to the question of the existence of a regularization) the action of corresponding solutions is the same. All that is still needed for establishing a classical equivalence is to prove that gauge orbits are sent to gauge orbits bijectively. This cannot be quite the case, since in the case of the cubic theory the gauge orbits are not well defined. Nonetheless, we can check these statements at the linearized level, where the cubic theory does not suffer from any obvious problems. We find in 3.3 that the correspondence indeed holds at that level. This also implies that the cohomologies around equivalent solutions are the same.

The cubic theory was already presented in section 2. Let us now turn to the non-polynomial theory, due to Berkovits [8]. In this theory picture changing operators are avoided altogether by working in the large Hilbert space. This enables one to work with a string field of ghost and picture number zero in the NS sector. The doubling of the degrees of freedom is compensated by a novel gauge symmetry. Again, the action can be written as a sum,

$$S = S_{\text{NS}} + S_R. \tag{3.1a}$$

The NS part of the action was given in [8],

$$S_{\text{NS}} = \frac{1}{2} \oint \left(e^{-\Phi} Q e^{\Phi} e^{-\Phi} \eta_0 e^{\Phi} - \int_0^1 dt \Phi [e^{-t\Phi} \eta_0 e^{t\Phi}, e^{-t\Phi} Q e^{t\Phi}] \right), \tag{3.1b}$$

where the integral \oint represents integration in the large Hilbert space.⁴

It is not easy to add the Ramond sector to the non-polynomial theory. The equations of motion were found in [8], but it seems that they cannot be derived from a covariant action. In [18], the difficulty with the Ramond sector was attributed to a self duality property of the string field⁵. It was suggested there, in analogy with the treatment of the type IIB self dual RR form, to introduce another string field and impose a self duality constraint between the two Ramond string fields. The Ramond part of the action is then,

$$S_R = -\frac{1}{2} \oint e^{-\Phi} Q \Xi e^{\Phi} \eta_0 \Psi, \tag{3.1c}$$

where Ξ and Ψ are the Ramond string fields. The action should be supplemented with the constraint,

$$e^{-\Phi} Q \Xi e^{\Phi} = \eta_0 \Psi, \tag{3.1d}$$

which should be imposed only after deriving the equations of motion.

⁴A novel representation of this action was given in [22], $S_{\text{NS}} = -\oint \int_0^1 dt \eta A_t A_Q$, where, A_{∇} stands for $e^{-t\Phi} \nabla e^{t\Phi}$ and ∇ is an arbitrary derivation of the star product. Here, ∇ can represent one of the odd canonical derivations Q and η_0 , as well as the variation δ or the derivative with respect to the parameter t , ∂_t . To get from (3.1b) to this form one has to rewrite the former as, $S_{\text{NS}} = \int_0^1 dt \oint \left(\partial_t (A_Q A_{\eta}) - A_t [A_{\eta}, A_Q] \right)$ and use several times the identity, $F_{\nabla_1 \nabla_2} \equiv \nabla_1 A_{\nabla_2} - (-)^{\nabla_1 \nabla_2} \nabla_2 A_{\nabla_1} + [A_{\nabla_1}, A_{\nabla_2}] = 0$.

⁵The Ramond string field contains a massless chiral fermion.

The equations of motion are now,

$$\eta_0(e^{-\Phi}Qe^{\Phi}) + \frac{1}{2}[\eta_0\Psi, e^{-\Phi}Q\Xi e^{\Phi}] = 0, \quad (3.2a)$$

$$\eta_0(e^{-\Phi}Q\Xi e^{\Phi}) = 0, \quad (3.2b)$$

$$Q(e^{\Phi}\eta_0\Psi e^{-\Phi}) = 0. \quad (3.2c)$$

Taking the constraint (3.1d) into consideration this set of equations reduces to,

$$\eta_0(e^{-\Phi}Qe^{\Phi}) + (\eta_0\Psi)^2 = 0, \quad (3.3a)$$

$$Q(e^{\Phi}\eta_0\Psi e^{-\Phi}) = 0. \quad (3.3b)$$

3.1 The mapping

For the NS sector, the mapping is given by

$$\tilde{A} = e^{-\Phi}Qe^{\Phi}. \quad (3.4)$$

We append this mapping with the Ramond counterpart,

$$\alpha = i\eta_0\Psi. \quad (3.5)$$

In these variables the equations of motion (3.3) take the form,

$$\eta_0\tilde{A} - \alpha^2 = 0, \quad (3.6a)$$

$$Q\alpha + [\tilde{A}, \alpha] = 0. \quad (3.6b)$$

These two equations should be appended with the consistency conditions that follow from the definitions (3.4) and (3.5),

$$Q\tilde{A} + \tilde{A}^2 = 0, \quad (3.6c)$$

$$\eta_0\alpha = 0. \quad (3.6d)$$

The last equation implies that α is defined in the small Hilbert space. However, the introduction of a non-trivial Ramond field implies that \tilde{A} is no longer a member of the small Hilbert space, as can be read from (3.6a). To remedy this problem we define,

$$A = \tilde{A} - \xi\alpha^2. \quad (3.7)$$

This string field does live in the small Hilbert space as a result of the definition (3.6a) and the commutation relation (1.3c).

Now, both variables live in the small Hilbert space as a result of (3.6a) and (3.6d) and the role of the equations of motion is played by (3.6b) and (3.6c). Rewriting these equations in terms of A, α we get exactly the equations of motion of the cubic theory (2.4). To that end, one has to use the nilpotency property (1.6d) as well as the fact that ξ is a midpoint insertion. To summarize, our mapping is given by,

$$A = e^{-\Phi}Qe^{\Phi} + \xi(\eta_0\Psi)^2, \quad (3.8a)$$

$$\alpha = i\eta_0\Psi. \quad (3.8b)$$

We discuss why the NS sector mapping of (3.8a) has to be modified in appendix B.

For the inverse mapping we define,

$$\Phi = PA, \tag{3.9a}$$

$$\Psi = -i\xi\alpha. \tag{3.9b}$$

These definitions are enough for showing that the equations of motion are invariant. However, since we are also interested in proving the invariance of the action, we also add the definition,

$$\Xi = -iP\alpha. \tag{3.9c}$$

One can check that the definitions of Ψ and Ξ are consistent with the constraint (3.1d).

Composing the maps (3.8) and (3.9) one gets,

$$A_{\text{new}} = e^{-\Phi} Q e^{\Phi} - \xi\alpha^2 = (1 - PA)Q(PA) - \xi\alpha^2 \tag{3.10a}$$

$$= A - P(QA + A^2) - \xi\alpha^2 = A + PX\alpha^2 - \xi\alpha^2 = A,$$

$$\alpha_{\text{new}} = \eta_0 \xi \alpha = \alpha. \tag{3.10b}$$

In (3.10a) we used the nilpotency of P (1.6c), the equation of motion (2.4a), as well as the OPE (1.6a), while in (3.10b) we used the fact that α is defined in the small Hilbert space.

In the opposite direction one gets,

$$e_{\text{new}}^{\Phi} = 1 + P(-Qe^{-\Phi}e^{\Phi} - \xi\alpha^2) = QPe^{-\Phi}e^{\Phi} = e^{-QP\Phi}e^{\Phi}, \tag{3.11a}$$

$$\Psi_{\text{new}} = \xi\eta_0\Psi = \Psi - \eta_0\xi\Psi, \tag{3.11b}$$

where in (3.11a) we first used the OPE (1.6e) and then we used (1.6c) and (1.3a). In this direction the composition of the transformations gives the identity operator only for a specific gauge choice for Ψ . Otherwise, it gives a gauge equivalent string field as will be shown in 3.3.

3.2 The action

We want to prove that the action of solutions is the same in both theories. For the NS part, this was shown to hold in [10]. In fact, what was proven there is the following: Given the mapping (3.9a), the values of the actions (2.2b) and (3.1b) are the same. For this proof the equations of motion were not used. Hence, we can use it here, regardless of the fact that the equations of motion of the NS sector are modified by Ramond terms.

To complete the proof we now show that in both theories the Ramond part of the action of an arbitrary solution is zero. For the cubic theory this results from the fact that both terms of the Ramond part of the action are proportional to α^2 and hence to the star product of α with the α equation of motion (2.4b). For the non-polynomial theory, (3.2b) implies that the Ramond part of the action integrand is annihilated by η_0 . Hence, its large-Hilbert-space integral is zero.

While we proved that the Ramond sector poses no new problems for the equality of the action, one should remember that for the proof of equality in the NS sector, the existence of an adequate regularization should be assumed [10]. We again assume that such a regularization exists.

3.3 Gauge transformations

As we already stressed, the finite gauge transformation of the cubic theory is not well defined. Hence, strictly speaking, there is no way to match gauge orbits between the two theories. Nevertheless, when restricted to linearized gauge transformations, all expressions make sense. We would therefore like to study the linearized gauge transformations, assuming that the action (2.2) is some sort of a singular limit of a well behaved cubic theory. Since the singularity of the cubic theory does not express itself at the linearized level, we expect that, at that level, the formal equivalence that we study works.

For the gauge transformations on the side of the non-polynomial theory, one should distinguish between the gauge symmetries of the action (3.1) and that of the equations of motion (3.3), which is obtained after the use of the constraint (3.1d). For the action, the gauge symmetries are,

$$\delta e^\Phi = e^\Phi \eta_0 \Lambda_1 + Q \Lambda_0 e^\Phi, \quad (3.12a)$$

$$\delta \Psi = \eta_0 \Lambda_{\frac{3}{2}} + [\Psi, \eta_0 \Lambda_1], \quad (3.12b)$$

$$\delta \Xi = Q \Lambda_{-\frac{1}{2}} + [Q \Lambda_0, \Xi], \quad (3.12c)$$

where the four gauge string fields are labeled by their picture numbers. This symmetry is consistent with the constraint (3.1d). With this constraint imposed, the gauge transformation generated by $\Lambda_{-\frac{1}{2}}$ becomes trivial. On the other hand, on the constraint surface there is an enhancement of symmetry, i.e., a new gauge string field $\Lambda_{\frac{1}{2}}$ generates gauge transformations on this surface [18]. The gauge symmetry takes the form,⁶

$$\delta e^\Phi = e^\Phi (\eta_0 \Lambda_1 - [\eta_0 \Psi, \Lambda_{\frac{1}{2}}]) + Q \Lambda_0 e^\Phi, \quad (3.13a)$$

$$\delta \Psi = \eta_0 \Lambda_{\frac{3}{2}} + [\Psi, \eta_0 \Lambda_1] + Q \Lambda_{\frac{1}{2}} + [e^{-\Phi} Q e^\Phi, \Lambda_{\frac{1}{2}}]. \quad (3.13b)$$

Suppose now that we map a solution of the cubic theory to the non-polynomial one and that this cubic solution is modified by a gauge transformation. This induces the following transformation on the side of the non-polynomial theory,

$$\delta \Phi = P \delta A = P(Q \Lambda + [A, \Lambda] + X[\alpha, \chi]), \quad (3.14a)$$

$$\delta \Psi = -i \xi \delta \alpha = -i \xi (Q \chi + [\alpha, \Lambda] + [A, \chi]). \quad (3.14b)$$

The map (3.9a) together with the nilpotency of P (1.6c) implies that

$$e^\Phi = 1 + \Phi = 1 + P A, \quad (3.15a)$$

$$\delta e^\Phi = \delta \Phi. \quad (3.15b)$$

Now, define

$$\Lambda_0 = -P \Lambda, \quad \Lambda_{\frac{1}{2}} = i \xi \chi, \quad \Lambda_1 = \xi \Lambda, \quad \Lambda_{\frac{3}{2}} = -i \tilde{\xi} X \chi. \quad (3.16)$$

Here, $\tilde{\xi}$ in the definition of $\Lambda_{\frac{3}{2}}$ represents a ξ insertion at any arbitrary point other than the midpoint, in order to avoid singularities from the OPE of X and ξ . Alternatively, we can

⁶Note that Λ_0 here is $-\Lambda_Q$ of [10].

take the normal ordered product $:\xi X:$. Since $\Lambda_{\frac{3}{2}}$ appears only in the combination $\eta_0\Lambda_{\frac{3}{2}}$, the point at which $\tilde{\xi}$ is inserted is of no consequence. With these gauge string fields the transformation (3.13) takes the form

$$\begin{aligned} \delta\Phi &= (1+PA)(\eta_0\xi\Lambda - [\eta_0\xi\alpha, \xi\chi]) - PQ\Lambda(1+PA) \\ &= (1+PA)(\Lambda - [\alpha, \xi\chi]) + (PQ\Lambda - \Lambda)(1+PA) \\ &= \Lambda + P\Lambda\Lambda + \xi[\alpha, \chi] + PQ\Lambda - \Lambda - P\Lambda\Lambda, \end{aligned} \tag{3.17a}$$

$$\begin{aligned} \delta\Psi &= -i\eta_0\tilde{\xi}X\chi - i[\xi\alpha, \eta_0\xi\Lambda] + iQ\xi\chi + i[(1-PA)Q(1+PA), \xi\chi] \\ &= -iX\chi - i\xi[\alpha, \Lambda] + iX\chi - i\xi Q\chi - i\xi[A, \chi]. \end{aligned} \tag{3.17b}$$

These transformations coincide with those of (3.14). Hence, a gauge transformation of the cubic theory induces a gauge transformation of the non-polynomial theory.

Let there now be two gauge equivalent solutions of the non-polynomial theory. There are four gauge string fields relating these two solutions, $\Lambda_0, \Lambda_{\frac{1}{2}}, \Lambda_1, \Lambda_{\frac{3}{2}}$. It is easy to see that Λ_0 and $\Lambda_{\frac{3}{2}}$ do not induce any variation of A and α . As for $\Lambda_{\frac{1}{2}}$ and Λ_1 , one can see that they induce a gauge transformation, where the gauge string fields in the side of the cubic theory are given by,

$$\chi = -i\eta_0\Lambda_{\frac{1}{2}}, \quad \Lambda = \eta_0\Lambda_1 + [\alpha, \chi]. \tag{3.18}$$

The proof is similar to the above, even if somewhat longer, and makes use of the equation of motion (2.4b), the OPE (1.6d), the relation (1.3b) and the graded Jacobi identity⁷ for the fields α, χ and A ,

$$[A, [\alpha, \chi]] - [\alpha, [\chi, A]] + [\chi, [A, \alpha]] = 0. \tag{3.19}$$

Finally, we have to show that mapping a solution of the non-polynomial theory to the cubic theory and then back to the non-polynomial one results in a solution, which is gauge equivalent to the original one. The opposite assertion is trivial, since as we showed, composing the mappings in the opposite order results in the identity mapping. Now, we have to use the finite form of the gauge transformation, since the original solution and the one obtained after the mappings are by no means infinitesimally close. There is no problem in working with the finite gauge transformation, since we now consider the side of the non-polynomial theory, where the finite gauge transformation is well defined. The expressions for the fields after the mappings are given by (3.11). It is clear that we do not have to exponentiate the full linearized gauge symmetry (3.13). All that is needed is to consider the following non-zero gauge fields,

$$\Lambda_0 = -P\Psi, \quad \Lambda_{\frac{3}{2}} = -\xi\Psi. \tag{3.20}$$

The finite gauge transformation generated by these fields gives exactly (3.11).

We can now conclude that (on-shell and up to the problems in the Ramond sector of the cubic theory) gauge orbits in one theory correspond to gauge orbits in the other theory. Also, the fact that the equations of motion and gauge symmetries are mapped to each other in both directions implies that the same holds also for their linearized versions. Hence, both theories have the same cohomologies around solutions.

⁷Recall that $[\cdot, \cdot]$ is the graded commutator in our notations (1.1).

4 The equivalence for general D-brane configurations

The general D-brane system introduces the need for Chan-Paton factors, as well as the choice of sectors (NS \pm / R \pm) that enter into each entry of the Chan-Paton matrix.⁸ The study of the possible Chan-Paton factors and NS/R sectors is nothing but the classification of possible open string theories.⁹ For this classification we need to impose the requirements of mutual locality and closure of all the OPE's, as well as the consistency of the interaction with closed strings. The requirement of local OPE's of all fields involved, implies that the NS $-$ sector cannot exist in the same Chan-Paton entry with Ramond fields and that the R \pm fields are also mutually exclusive. Another requirement is the closure of the OPE, which implies that the NS $+$ sector is always present. Hence (at any given Chan-Paton entry) one may have either only the NS $+$ sector, or the NS $+$ sector with NS $-$ or with one of the R sectors. The combination of NS $+$ with either of R \pm can be realised on the D-brane and on the \bar{D} -brane. The NS \pm case is realised on the non-BPS D-brane. The pure NS $+$ case can also be realised [24].

The introduction of Chan-Paton factors into string field theory is easy. One simply tensors each string field with the appropriate Chan-Paton matrix and adds to the definition of the integral also a normalized trace over the Chan-Paton space [11]. The operators of the theories, namely Q , η_0 and Y_{-2} are not affected and can be thought of as being multiplied by the identity matrix in the Chan-Paton space.

Adding the NS $-$ sector to the NS $+$ one, when working with a non-BPS D-brane (or at an off-diagonal entry of the Chan-Paton matrix that represents strings stretching between a D-brane and a \bar{D} -brane), can be achieved by tensoring the previous structure also with “internal Chan-Paton” (two by two) matrices and adding to the definition of the integral also a normalized trace over this sector.¹⁰ This structure was first introduced for the non-polynomial theory in [11], where it was shown that the NS $+$ sector should be tensored with the two by two identity matrix and the NS $-$ sector should be tensored with the Pauli matrix σ_1 . The gauge string fields for these sectors are tensored with σ_3 and $i\sigma_2$ respectively and the operators Q and η_0 are also tensored with σ_3 . For the cubic theory this structure was introduced in [12], where it was shown that the roles of the string fields and the gauge fields are reversed, e.g., the NS $-$ string field A_- gets the $i\sigma_2$ factor and the NS $-$ gauge field gets the σ_1 . The kinetic operator Q retains the σ_3 factor that should also be granted to Y_{-2} .

This similarity in the structures of describing the NS \pm sectors in both theories, makes the generalization of the mapping to the NS $-$ sector straightforward. Indeed, the generalization of the mapping for the NS $-$ sector and for the case of Chan-Paton factors was already given in [10]. All that is needed is to tensor ξ and P with σ_3 (and with the identity matrix in the genuine Chan-Paton space) and the mappings work.

⁸Here, we consider explicitly the ten dimensional, flat, Poincaré-invariant cases. Generalization to the case with lower dimensional D-branes should be simple.

⁹See [23] for a thorough discussion on open string classification.

¹⁰The internal Chan-Paton matrices compensate for the opposite Grassmann parity of the two sectors and restrict the interaction terms only to those that respect the GSO parity.

Now, we want to consider the case of adding also R_{\pm} sectors in various entries of the Chan-Paton matrix. As mentioned in the introduction, the Ramond string fields are odd, just like the NS+ string field. Furthermore, we never deal with both Ramond sectors or with a Ramond sector together with an NS- sector. Hence, there is no need to append any of the Ramond sectors with internal Chan-Paton factors. Nevertheless, in the case where some other entries of the Chan-Paton matrix contain an NS- sector, it is possible, just for the sake of a uniform description of the whole Chan-Paton matrix, to append internal Chan-Paton factors also to the Ramond sector string fields. In such a case one should define an internal Chan-Paton factor for the Ramond string field, σ_R . The σ_3 factors on A , Q and Y_{-2} imply that σ_R should square to the identity matrix and commute with σ_3 . Hence, one should either choose $\sigma_R = \sigma_3$ or $\sigma_R = \mathbf{1}$.

The same can be done for the non-polynomial theory. There, the string fields Ξ and Ψ would have to be appended with σ_R that obeys the same conditions as in the case of the cubic theory. Now, commutativity with σ_3 should be implied due to its presence in Q and η_0 .

For our mapping to work in these cases as well, one should make the opposite choice for σ_R in the two theories, since σ_3 appears with an odd power everywhere in our mappings (3.8) and (3.9). The most natural definition would be to define $\sigma_R = \mathbf{1}$ in the non-polynomial theory and $\sigma_R = \sigma_3$ in the cubic theory. Then, all the string fields (other than the NS- ones) at any of the two theories, are appended with the same factor. The same uniformity holds also for the gauge string fields. All the good properties of our mappings are maintained.

5 A cubic action with two fields in the Ramond sector

Establishing the correspondence between the cubic theory (2.2) and the non-polynomial theory (3.1) including the constraint (3.1d), one may ask whether there is some sort of an extension of the mapping for the unconstrained non-polynomial theory as well. For such a mapping to exist, we have to consider a modification of the cubic theory with two Ramond fields. In fact, this is a natural avenue from the perspective of the cubic theory as well, as we explain below.

Furthermore, the fact, discussed above, that the Ramond sector of the cubic theory is ill-defined, calls for a refined, mid-point-insertion-free formulation. Recall that the problems of the formalism originate from the mid-point insertions in the definitions of the gauge transformations, not from the mid-point insertion in the definition of the action. In fact, when string fields with mid-point insertions are assumed to be outside the space of allowed string fields, the use of mid-point insertions in the definition of the action does not lead to any problems. We refer again to [7] for further discussion on this issue.

The modified cubic theory solved the problems of the original cubic theory by working with NS fields in the “neutral” (zero) picture. This cannot be imposed on the Ramond fields, since they carry half-integer picture numbers. The closest one can get is to use a $\pm\frac{1}{2}$ picture. Of these two options, the “natural” Ramond ($-\frac{1}{2}$) picture was selected. The fact that a non-zero string field is used implies that picture changing operators appear

in the equations of motion and gauge transformations. These operators must be inserted at the mid-point, leading to potential singularities of the theory. If two Ramond fields are allowed, one can write an action, whose equations of motion will not include picture changing operators other than the global Y_{-2} insertion. Of course, the physical theory has only one Ramond field. Hence, a constraint should be imposed. This constraint is bound to include picture changing operators, leading to the usual equations of motion (2.4). So one may be under the impression that nothing is gained. Still, the fact that the new action is equivalent (as we shall promptly see) to the unconstrained non-polynomial action may suggest that these two actions might have some meaning even before the constraint is imposed. Moreover, one might speculate that the new action can be useful for quantization and for generalizations of the theory.

From the above discussion one can immediately guess the action,¹¹

$$S = - \int Y_{-2} \left(\frac{1}{2} AQA + \frac{1}{3} A^3 + \frac{1}{2} \tilde{\alpha}Q\alpha + \frac{1}{2} A[\alpha, \tilde{\alpha}] \right), \quad (5.1)$$

with α having picture number $n_p(\alpha) = -\frac{1}{2}$ as before and $n_p(\tilde{\alpha}) = \frac{1}{2}$. From this action follow the equations of motion (omitting the global Y_{-2}),

$$QA + A^2 + \frac{1}{2}[\alpha, \tilde{\alpha}] = 0, \quad (5.2a)$$

$$Q\alpha + [A, \alpha] = 0, \quad (5.2b)$$

$$Q\tilde{\alpha} + [A, \tilde{\alpha}] = 0. \quad (5.2c)$$

It is interesting to notice that the three string fields can be unified in terms of a single string field simply by adding them. Define,

$$\hat{A} = A + \frac{\alpha + \tilde{\alpha}}{\sqrt{2}}, \quad (5.3)$$

and expand in terms of these constituents the natural action for \hat{A} ,

$$S = - \int Y_{-2} \left(\frac{1}{2} \hat{A}Q\hat{A} + \frac{1}{3} \hat{A}^3 \right). \quad (5.4)$$

Expanding the integrand one gets the integrand of the action (5.2a) together with some other terms. However, all other terms have wrong picture numbers and hence can be safely dropped out of the action.

The constraint we should impose is,

$$\tilde{\alpha} = X\alpha \iff \alpha = Y\tilde{\alpha}. \quad (5.5)$$

While both representations above are correct, the left one is more accurate in the sense that $\tilde{\alpha}$ is the string field with the actual mid-point X insertion. A genuine Y insertion is prohibited, since it would lead to singularities with the Y_{-2} insertion in the action.

¹¹The proofs of the various properties of the mapping between this action and the unconstrained non-polynomial action are quite similar to those of 3. Hence, we tend to be brief here.

Keeping this constraint in mind, we also prohibit explicit X insertions in α . Applying this constraint, the action and the equations of motion reduce to (2.2) and (2.4) respectively.

At the linearized level the action (5.1) is invariant under three independent gauge transformations,

$$\delta A = Q\Lambda, \quad \delta\alpha = Q\chi, \quad \delta\tilde{\alpha} = Q\tilde{\chi}. \quad (5.6)$$

However, only the first of these gauge transformations has an extension at the non-linearized level. Hence, the complete gauge invariance of the theory reads,

$$\delta A = Q\Lambda + [A, \Lambda], \quad (5.7a)$$

$$\delta\alpha = [\alpha, \Lambda], \quad (5.7b)$$

$$\delta\tilde{\alpha} = [\tilde{\alpha}, \Lambda]. \quad (5.7c)$$

This can be understood from the compact form of the action (5.4). This form implies that the gauge symmetry is,

$$\hat{A} \rightarrow e^{-\hat{\Lambda}}(Q + \hat{A})e^{\hat{\Lambda}}. \quad (5.8)$$

If one tries to substitute into \hat{A} any component, whose picture is non-zero, it will result in taking the string field \hat{A} out of the allowed picture-number range, $-\frac{1}{2} \leq p \leq \frac{1}{2}$. Hence, $\hat{\Lambda}$ should be restricted to have only the zero picture component Λ and the gauge transformation reduces (in its linearized form) to (5.7).

Imposing the constraint (5.5) leads to an enhancement of the gauge symmetry to (2.5) (modulo the consistency problem of this gauge transformation), in analogy with the situation in the non-polynomial theory. In fact, the absence of gauge symmetries for the fermionic string fields should be expected, if we indeed believe (and shortly prove) that this theory is equivalent to the non-constrained non-polynomial theory. This can be seen by counting degrees of freedom. The non-polynomial theory resides in the large Hilbert space, that has double the degrees of freedom of the small Hilbert space, due to the presence of the ξ_0 mode. In this space both Q and η_0 are trivial and hence gauge transformations based on them reduce the degrees of freedom by a half. For the boson field, this theory has the Λ_1 gauge symmetry that effectively implies that the degrees of freedom of the theory are isomorphic to those of the small Hilbert space. Then, on top of this gauge symmetry there is also the Λ_0 gauge symmetry that reduces the degrees of freedom of the theory in exactly the same way that Λ reduces those of the cubic theory. Note that we can no longer claim that this is a reduction by ‘‘a half’’, since in the small Hilbert space Q is no longer trivial. The two fermionic gauge symmetries of the theory, namely $\Lambda_{-\frac{1}{2}}$ and $\Lambda_{\frac{3}{2}}$ reduce to a half the degrees of freedom of Ξ and Ψ respectively, rendering them potentially equivalent to α and $\tilde{\alpha}$. Had the cubic theory had more gauge symmetry, its degrees of freedom could not have matched those of the non-polynomial one.

Our goal now is to find the mapping between the two theories. We propose the mapping,

$$\Phi = PA, \quad \Psi = -iP\tilde{\alpha}, \quad \Xi = -iP\alpha. \quad (5.9)$$

We have to show that under this mapping solutions of the equations of motion (5.2) are mapped to solutions of (3.2). From (5.9) we find that the l.h.s of (3.2a) is given by,

$$\eta_0(e^{-\Phi} Q e^{\Phi}) + \frac{1}{2}[\eta_0 \Psi, e^{-\Phi} Q \Xi e^{\Phi}] = -Y \left(Q A + A^2 + \frac{1}{2}[\alpha, \tilde{\alpha}] \right), \quad (5.10)$$

which vanishes in light of the equation of motion (5.2a). Then, we find,

$$e^{-\Phi} Q \Xi e^{\Phi} = -i\alpha, \quad (5.11)$$

where we used (1.6c). This implies that (3.2b) holds.

For calculating (3.2c), define

$$\hat{\alpha} \equiv -iY\tilde{\alpha}, \quad (5.12)$$

and evaluate,

$$Q(e^{\Phi} \eta_0 \Psi e^{-\Phi}) = Q((1 + PA)\hat{\alpha}(1 - PA)) = Q\hat{\alpha} + [A, \hat{\alpha}] - PQ[A, \hat{\alpha}] = 0, \quad (5.13)$$

where the last equality follows from (5.2c). The last manipulation can be criticized on the ground that a factor of Y was “hidden” in the definition of $\hat{\alpha}$. When considered explicitly this factor would produce divergences with the factors of P that multiply it. We can avoid this problem in one of two ways. One way is to recall that when we enforce the constraint relating α and $\tilde{\alpha}$ the later has an explicit factor of X multiplying it. We can declare that $\tilde{\alpha}$ has to have such an insertion even without the constraint. This does not change a priori the amount of degrees of freedom of $\tilde{\alpha}$ and solves the problem. Nonetheless, this resolution is not quite satisfactory since our aim was to obtain a theory, which at least a priori is free from explicit mid-point insertions over string fields. The other way is to rely on the fact that we have to regularize the mappings anyway, in order to produce a sensible action. Then, we can declare that obtaining the above result without any finite corrections is a symmetry principle for the regularization scheme. This resolution further constrains the needed regularization. However, we have more string fields now in our disposal, so it is plausible that a regularization exists.

For the inverse mapping we choose,

$$\alpha = ie^{-\Phi} Q \Xi e^{\Phi}, \quad (5.14a)$$

$$\tilde{\alpha} = i\eta_0 X \Psi, \quad (5.14b)$$

$$A = e^{-\Phi} Q e^{\Phi} + \frac{1}{2}\xi[\eta_0 \Psi, e^{-\Phi} Q \Xi e^{\Phi}]. \quad (5.14c)$$

The equations of motion of the non-polynomial theory (3.2) imply that A , α and $\tilde{\alpha}$ live in the small Hilbert space as they should. Next, we have to verify that the equations of motion (5.2) also follow from (3.2) and the mapping (5.14). The proof for (5.2a) and (5.2b) is straightforward. For the evaluation of the last equation (5.2c), define

$$\hat{\Psi} = iX\Psi. \quad (5.15)$$

We then get,

$$\begin{aligned} Q\tilde{\alpha} + [A, \tilde{\alpha}] &= Q\eta_0\hat{\Psi} + [e^{-\Phi}Qe^{\Phi} + \frac{1}{2}\xi[\eta_0\Psi, e^{-\Phi}Q\xi e^{\Phi}], \eta_0\hat{\Psi}] \\ &= Q\eta_0\hat{\Psi} + [(1 - \xi\eta_0)e^{-\Phi}Qe^{\Phi}, \eta_0\hat{\Psi}] = Q\eta_0\hat{\Psi} - \eta_0\xi Q\eta_0\hat{\Psi} = \xi\eta_0Q\eta_0\hat{\Psi} = 0, \end{aligned} \quad (5.16)$$

where in the second equality (3.2a) was used and in the next equality (3.2c) was used. Note, that similarly to what we had in (5.13), a singularity due to the collision of X and ξ is hidden in the definition of $\hat{\Psi}$. Here, we have no excuse for claiming that Ψ should always have a factor Y in its definition for cancelling the X that multiplies it. Hence, the only way out is to require the existence of a good regularization scheme. We stress again, that as in the previous cases where we relied on the existence of the regularization, i.e., in the evaluation of the action in [10] and in (5.13), we have neither an explicit form of the regularization nor a proof of its existence. We return to this point in section 6.

It is straightforward to see that composing the mapping (5.14) on (5.9) results in the identity mapping of the cubic theory. Suppose now that we compose the mappings in the opposite order. We expect to get a finite gauge transformation. In fact we get,

$$e_{\text{new}}^{\Phi} = e^{-QP\Phi}e^{\Phi}, \quad (5.17a)$$

as before, while for the fermionic fields we get,

$$\Psi_{\text{new}} = \Psi - \eta_0\xi\Psi, \quad (5.17b)$$

$$\Xi_{\text{new}} = Pe^{-\Phi}Q\xi e^{\Phi} = PQP e^{-\Phi}Q\xi e^{\Phi} = e^{-QP\Phi}(\Xi - QP\xi)e^{QP\Phi}. \quad (5.17c)$$

Exponentiating the various transformations of (3.12) in order to get the form of the finite gauge transformations one can see that (5.17) can be obtained by performing the following finite gauge transformations in the order they are written,

$$\Lambda_{\frac{3}{2}} = -\xi\Psi, \quad \Lambda_{-\frac{1}{2}} = -P\xi, \quad \Lambda_0 = -P\Phi. \quad (5.18)$$

Let there be two gauge equivalent solutions of the non-polynomial theory. The infinitesimal gauge transformations are generated by the four gauge fields Λ_p with $p = -\frac{1}{2}, 0, 1, \frac{3}{2}$. Considering each of these transformations and mapping it to the cubic theory we see that only Λ_1 induces a non-trivial transformation of the cubic string fields. This variation is given by (5.7), with the identification

$$\Lambda = \eta_0\Lambda_1. \quad (5.19)$$

We conclude that gauge equivalent configurations are mapped to gauge equivalent configurations in this direction. In the side of the cubic theory we have only one gauge transformation to consider. Substituting (5.7) into the mapping gives,

$$\delta e^{\Phi} = e^{\Phi}\Lambda - QP\Lambda e^{\Phi}, \quad \delta\Psi = [\Psi, \Lambda], \quad \delta\xi = [\xi, QP\Lambda]. \quad (5.20)$$

This is a gauge transformation in the side of the non-polynomial theory with the gauge string fields given by,

$$\Lambda_0 = -P\Lambda, \quad \Lambda_1 = \xi\Lambda, \quad \Lambda_{-\frac{1}{2}} = \Lambda_{\frac{3}{2}} = 0. \quad (5.21)$$

We can now conclude that, when no constraints are imposed, the mapping of gauge orbits between the two-Ramond-field cubic theory and the unconstrained non-polynomial theory is bijective.

The proof of the equality of the action of solutions in both theories is very similar to what we presented in 3.2. Again, we prove that the Ramond-sector contribution to the action is zero for solutions in both theories. For the non-polynomial theory the proof only used (3.2b), which does not depend on the constraint. Hence, there is nothing new to prove here. For the cubic theory the Ramond part of the action integrand is bi-linear in α and $\tilde{\alpha}$, which implies that it is equal to the star product of α with its equation of motion, as well as to the star product of $\tilde{\alpha}$ with its equation of motion. Any one of these equalities is enough to conclude that the action of solutions gets no contribution from this sector.

We proved the equivalence of gauge orbits, which also implies the identity of the cohomologies around solutions as well as the equality of the action of corresponding solutions. Hence, we conclude that the two theories are classically equivalent.

Finally, let us illustrate that the constraints one has to impose on both theories are equivalent. Starting at the cubic theory we have to impose (5.5). This immediately implies,

$$\eta_0 \Psi = -iY \tilde{\alpha} = -i\alpha = e^{-\Phi} Q \Xi e^{\Phi}, \tag{5.22}$$

which is just the constraint of the non-polynomial theory (3.1d). The other direction is as straightforward. The equivalence of the constraints together with the other results of this section give an “alternative” derivation of all the results of section 3, since imposing the constraints on both theories reduce them to the theories studied there.

6 Conclusions

In this work we proved the inconsistency of the modified cubic superstring field theory. The inconsistency stems from collisions of picture changing operators in the finite form of its Ramond-sector gauge transformations. This state of affairs implies that the cubic theory should either be abandoned or modified. We believe that the latter is the more sensible option, for two reasons. The first, stressed throughout this paper, is the formal equivalence between this theory and the non-polynomial theory. The second reason is the success of this theory in describing the NS sector. In particular vacuum solutions and dynamical tachyon condensation were studied both numerically and analytically, with very impressive results [10, 25–29]. This can be compared to Witten’s superstring field theory that failed to reproduce any such results [30]. We believe that had the cubic theory been completely wrong, it would have not produced these results even in the NS sector. Hence, we need a theory whose NS part reduces to the NS sector of the cubic theory in some limit. This theory should also consistently include the Ramond sector. A possible modification for the cubic theory was recently proposed using non-minimal sectors [7, 31]. It seems, however, that it cannot solve the consistency problem of the Ramond sector [7]. In section 5, we introduced a first step towards a different sort of a modification. There, the Ramond sector was doubled in order to avoid mid-point insertions of picture changing operators on string fields. The doubling of the Ramond sector kills supersymmetry. In order to restore it,

one might consider doubling the NS sector as well. One might even speculate that such a doubling may be useful for constructing closed superstring field theories, especially in light of [32].

At any rate, the cubic theory with doubled Ramond sector is by itself not satisfactory, since it does not have the correct amount of degrees of freedom. Following the example of [18], we tried to resolve this problem by introducing a constraint. It seems, however, that such a constraint is bound to include explicit mid-point insertions, which we have to avoid. Hence, we have to look for another sort of resolution. An appealing possibility is to further enlarge the field content as well as the gauge symmetry, such that the final amount of degrees of freedom is reduced to the correct one. We currently study this possibility and generalizations thereof.

The second issue studied in this work is the formal equivalence of the cubic and non-polynomial theories. All the properties studied in this work were shown to be invariant under our mappings, supporting the equivalence. However, there is one more invariant that one might wish to consider. This is the boundary state constructed from the solution [15] (see also [33–37]). To study this in the context of our equivalence one would first have to combine some ideas from [38] and [15], in order to define boundary states for the supersymmetric theories. We currently study this subject.

It might seem strange that the Ramond parts of our mappings (3.8b) and (3.9b), include factors of i . One might worry that our construction is inconsistent with the reality condition of the string field. In fact, it is the other way around. Recall that, in the Ramond sector, the coefficient fields are Grassmann odd. The reality condition is obtained by composing Hermitian conjugation and BPZ conjugation. While the first inverts the order of insertions in the usual way, the second does not change the formal Grassmann order [39]. Hence, the reality of the string field α implies that the string field $\xi\alpha$ is imaginary. The i 's take care just of that.

Much of the recent renewal of interest in string field theory is due to Schnabl's solution [16] and subsequent work [40–58]. Having explicit analytical expressions for string field theory solutions evoked the realisation that these solutions can be formally represented as pure gauge solutions. In the bosonic theory, solutions can be written using singular gauge string fields [59]. For the cubic superstring field theory in the NS sector, the equivalence of [10] provides a formal gauge form for the solutions. The “gauge string field” is formal in this case since it resides in the large Hilbert space. The extension of the equivalence to the Ramond sector presented here does not seem to define a solution with a form of a formal gauge solution. This statement, however, is ill-defined, since the finite gauge transformation of the Ramond sector does not exist for the cubic theory. One might suspect that in a well defined refinement of the cubic formalism it would be possible to write solutions as formal gauge ones. However, in the cubic formalism with two Ramond fields that we introduced in 5, one sees that the gauge transformation of the Ramond fields (5.14) is zero for a solution with zero Ramond fields. Thus, one cannot get a solution with non-trivial Ramond fields as a gauge solution around the vacuum. This can be traced to the fact that we have no gauge symmetry in this theory whose generators are fermionic. There is also no supersymmetry in this theory. It might be the case that in a more physical refinement of the cubic theory there will be a natural way for writing solutions as formal gauge ones.

While studying the mappings between the cubic and non-polynomial two-Ramond-field theories, we got twice expressions, which were formally of the form of a zero times a divergence that came from a mid-point collision of operators. This implies that a regularization is needed in which these expressions could be consistently set to zero. We thus have two requirements from a consistent regularization, on top of the requirement that we had from calculating the NS action [10]. On the other hand, we also have two more string fields, i.e., the Ramond ones and two more mappings (in each direction) to modify for defining the regularizations. Hence, the existence of a sound regularization is as plausible as it is for the NS case. We would like to stress that even regardless of the issue of singularities, a regularization is desirable, since the mappings we introduced include various mid-point insertions on string fields. Mid-point insertions on string fields are highly constrained and a formulation that avoids them altogether would be more reliable [7].

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A Quantum number tables

In this appendix we present the sector (R/NS), ghost number n_g , picture number n_p and parity of the string fields and string gauge fields. Those of the modified cubic theory are presented in table 1 while those of the non-polynomial theory are presented in table 2. As the tables imply, string fields are odd and gauge string field are even for the cubic theory, while the opposite is true for the non-polynomial theory. Also, in table 3 we present the quantum numbers of the operators that compose the string fields.

string field	R/NS	n_g	n_p	parity
A	NS	1	0	1
α	R	1	$-\frac{1}{2}$	1
Λ	NS	0	0	0
χ	R	0	$-\frac{1}{2}$	0

Table 1. Modified cubic theory: all string fields live in the small Hilbert space.

string field	R/NS	n_g	n_p	parity
Φ	NS	0	0	0
Ψ	R	0	$\frac{1}{2}$	0
Ξ	R	0	$-\frac{1}{2}$	0
$\Lambda_{-\frac{1}{2}}$	R	-1	$-\frac{1}{2}$	1
Λ_0	NS	-1	0	1
$\Lambda_{\frac{1}{2}}$	R	-1	$\frac{1}{2}$	1
Λ_1	NS	-1	1	1
$\Lambda_{\frac{3}{2}}$	R	-1	$\frac{3}{2}$	1

Table 2. Non-polynomial theory: string fields live in the large Hilbert space. The subscripts of the gauge fields represent their picture number.

field	h	n_g	n_p	parity
∂X^μ	1	0	0	0
ψ^μ	$\frac{1}{2}$	0	0	1
b	2	-1	0	1
c	-1	1	0	1
ξ	0	-1	1	1
η	1	1	-1	1
$e^{q\phi}$	$-\frac{q(q+2)}{2}$	0	q	$q \pmod{2}$
J_B	1	1	0	1
P	0	-1	0	1
X	0	0	1	0
Y	0	0	-1	0
X_2	0	0	2	0
Y_{-2}	0	0	-2	0

Table 3. The conformal weight h , ghost number n_g , picture number n_p and parity of some conformal fields.

B Why should the bosonic mapping be modified

The extra piece in the r.h.s of (3.8a) may seem strange. Here, we want to discuss its origin.

The equation of motion (2.4a) is potentially problematic, since it involves the mid-point operator insertion X . It implies that either A or α are allowed such an insertion. If we assume that A is not allowed to have mid-point insertions, it follows that either $\alpha^2 = 0$, which is too restrictive, or that α^2 contains a factor of Y that cancels the X . For that to be the case it should be possible to write

$$\alpha = \mathcal{O}^1 \alpha^{(1)} + \mathcal{O}^2 \alpha^{(2)}, \tag{B.1}$$

where $\mathcal{O}^{1,2}$ are zero weight primaries inserted at the mid-point, whose ghost numbers are opposite and whose picture numbers sum up to -1 . These operators should also obey the OPE's

$$\mathcal{O}^1 \mathcal{O}^1 \sim 0, \quad \mathcal{O}^2 \mathcal{O}^2 \sim 0, \quad \mathcal{O}^1 \mathcal{O}^2 \sim Y. \tag{B.2}$$

We failed to find such operators, both in the NS sector and in the Ramond sector. In particular the choice $\mathcal{O}^1 = Y$, $\mathcal{O}^2 = 1$ does not obey the above OPE's.¹²

Let us now assume that the mid-point insertion originates from A . We can decompose A as

$$A = A_0 + \xi A_{-1}, \tag{B.3}$$

where the subscript represents the picture-number. Substituting into (2.4a) and collecting the coefficients of X , ξ and 1, that should separately vanish, we get,

$$QA_0 + A_0^2 = 0, \tag{B.4a}$$

$$A_{-1} + \alpha^2 = 0, \tag{B.4b}$$

$$QA_{-1} + [A_0, A_{-1}] = 0. \tag{B.4c}$$

The equation for the Ramond field (2.4b) reduces now to,

$$Q\alpha + [A_0, \alpha] = 0. \tag{B.4d}$$

Equation (B.4a) states that A_0 is a flat connection, while (B.4c) and (B.4d) imply respectively that A_{-1} and α are ‘‘covariantly constant’’ with respect to A_0 . Equation (B.4b) is a constraint equation relating A_{-1} and α . With this constraint imposed, (B.4c) follows from (B.4d). At this stage, one may think that A_{-1} can be discarded altogether, since it is fixed by α . This is not quite the case, since from its definition (B.3) we see that its contribution to A lives in the large Hilbert space. Since A itself lives in the small Hilbert space, it follows that this contribution is needed in order to cancel other terms in A_0 that live in the large Hilbert space.

Comparing (B.3) to (3.8a), we see that

$$A_0 = e^{-\Phi} Q e^{\Phi}. \tag{B.5}$$

¹²One can imagine relaxing the first two OPE's in (B.2), so as to allow a larger algebra of mid-point insertion operators. We did not manage to find a way to construct that either.

This is the expression for A that one gets without the Ramond sector. Adding the Ramond sector implies that this expression does not live anymore in the small Hilbert space. The resolution is to define A_{-1} as above, in order to bring A back to the small Hilbert space. This amounts to using the mapping (3.8).

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